

Can $Y(4140)$ be a $c\bar{c}s\bar{s}$ tetraquark ?

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Abstract

In this exploratory study the spectrum of tetraquarks of type $c\bar{c}s\bar{s}$ is calculated within a simple quark model with chromomagnetic interaction and effective quark masses extracted from meson and baryon spectra. It is tempting to see if this spectrum can accommodate the resonance $Y(4140)$, observed by the CDF collaboration, but not yet confirmed. The results seem to favour the $J^{PC} = 1^{++}$ sector where the coupling to the VV channel is nearly as small as that of $X(3872)$, when described as a $c\bar{c}q\bar{q}$ tetraquark. This suggests that $Y(4140)$ could possibly be the strange partner of $X(3872)$, in a tetraquark interpretation. However the sector $J^{PC} = 0^{++}$ cannot entirely be excluded. This work questions the practice of extracting effective quark masses containing spin independent contributions, from mesons and baryons, to be used in multiquark systems as well.

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I. INTRODUCTION

The CDF Collaboration [1] has recently observed a narrow structure in the $J/\psi\phi$ mass spectrum of $B^+ \rightarrow J/\psi\phi K^+$ decays, which has been named Y(4140). Its mass and decay width are $M = 4143.0 \pm 2.9(\text{stat}) \pm 1.2(\text{syst}) \text{ MeV}/c^2$ and $\Gamma = 11.7_{-5.0}^{+8.3}(\text{stat}) \pm 3.7(\text{syst}) \text{ MeV}/c^2$ respectively, which suggest that its structure does not fit conventional expectations for charmonium states. The CDF Collaboration expects that the $J/\psi\phi$ final state, with positive C-parity and two $J^{PC} = 1^{--}$ vector mesons (VV), is a good candidate for an exotic meson search. This resonance is well above the threshold for open charm decay $D_s^+ D_s^-$ at 3936.68 MeV and a charmonium $c\bar{c}$ with this mass would decay into an open charm pair predominantly and have a small branching fraction into $J/\psi\phi$ [2]. The mass of Y(4140) is below the threshold of the decay channel $D_s^{*+} D_s^{*-}$ at 4224.6 MeV, and not far above the $J/\psi\phi$ threshold at 4116.4 MeV.

More recently the Belle Collaboration reported preliminary results on Y(4140) [3]. No significant signal was found but their efficiency is low for the mass of Y(4140). The upper limit on the production rate $\mathcal{B}(B^+ \rightarrow Y(4140)K^+, Y(4140) \rightarrow J/\phi)$ is 6×10^{-6} at 90% C.L. This upper limit is lower than the central value of the CDF measurement $(9 \pm 3.4 \pm 2.9) \times 10^{-6}$ [1] which is thus considered not to contradict the CDF measurement.

The Belle Collaboration also searched for Y(4140) in the $J/\psi\phi$ mass spectrum of the two-photon process $\gamma\gamma \rightarrow J/\psi\phi$ [3]. Again, the efficiency was low and no signal was reported. In exchange, evidence was found for a new narrow structure at 4.35 MeV and width 13.3 MeV, with a statistical significance of about $\sim 3.5\sigma$ in the $J/\psi\phi$ mass spectrum. This resonance was named X(4350).

As such, the present situation allows a new opportunity to look for exotics. The fashionable option of a $D_s^* \bar{D}_s^*$ molecule has been considered in Refs. [4–7] and the QCD sum rules in Ref. [8–10], where states with $J^{PC} = 0^{++}$ or 2^{++} are favoured. Let us note however that the Belle Collaboration measurement of a two-photon partial width disfavors the scenario of Y(4140) to be a $D_s^* \bar{D}_s^*$ molecule with $J^{PC} = 0^{++}$ or 2^{++} [3].

Prior to the observation of Y(4140) by the CDF Collaboration, predictions for tetraquarks $c\bar{c}s\bar{s}$ seen as diquark-antidiquark systems with various J^{PC} were made in a simple non-relativistic model including $\ell = 0$ and 1 partial waves in Ref. [11] and in a relativistic framework based on the quasipotential approach in Ref. [12]. In the latter, states with 0^{++}

and $1^{+\pm}$ acquired masses in the range 4.1 - 4.2 MeV.

We should also mention that the resonance $Y(4140)$ was studied as the second radial excitation of the P-wave charmonium χ_{cJ}'' ($J = 0$ and 1), looking at the hidden charm decay mode. The conclusion was that such a description is problematic [13].

Deciphering the nature of $Y(4140)$, if confirmed in the future, (presently the B-factories have a poor acceptance for $B \rightarrow KJ/\psi\phi$ in the desired range [14]), is a new challenge. Thus it is legitimate to consider the tetraquark interpretation without correlated quarks or antiquarks, and try to find out if the $Y(4140)$ fits into the spectrum of the $c\bar{c}s\bar{s}$ system. Most important, we search for the decay pattern given by this possible structure. For simplicity, we use the model of Ref. [15] which successfully describes the $X(3872)$ as a $c\bar{c}q\bar{q}$ tetraquark. In Ref. [15] it was shown that $X(3872)$ can be interpreted as an eigenstate of the chromomagnetic interaction, where the lowest 1^{++} has a dominant octet-octet component (0.9997) and a very small singlet-singlet component (0.026) which explains why this state decays with a very small width into $J/\psi + \rho$ or $J/\psi + \omega$, in agreement with the experimental value for the total width $\Gamma < 2.3$ MeV of $X(3872)$ [16], and that the $J/\psi +$ pseudoscalar channel is absent. As $Y(4140)$ is seen to be narrow and decays into two vector mesons we wonder whether or not the same mechanism can give an explanation of its small width, about 5 times larger than that of $X(3872)$, and similar to that of $X(4350)$, but considerably narrower than the decay width of every other X,Y or Z resonances.

The paper is organized as follows. In Sec. II we introduce the quark model used in this study. In Sec. III we recall the basis states in the direct meson-meson channel with emphasis on the charge conjugation quantum number. In Sec. IV we present the matrix elements of the Hamiltonian [15] for $J^{PC} = 0^{++}, 1^{++}, 1^{+-}$ and 2^{++} states. In Sec. V we show the calculated spectrum and discuss its features. The last section is devoted to conclusions. In Appendix A we derive the orthogonal transformation from the direct meson-meson channel to the exchange meson-meson channel for states 0^{++} , in Appendix B for states with 1^{++} and 1^{+-} and in Appendix C for states with 2^{++} . Appendix D is devoted to an attempt to dynamically derive effective masses in a standard constituent quark model in order to justify the simplicity of the present study and enlighten the choice of effective masses.

II. THE MODEL

This is an exploratory study, based on the simple model of Ref. [15] which can reveal the basic features of the $c\bar{c}s\bar{s}$ tetraquark, especially the structure of the wave functions. In the next section we introduce the relevant basis states in the color-spin space, including both the singlet-singlet channels and the octet-octet, simply called hidden color channels. There are no correlated quarks or diquarks, as in Ref. [11], for example.

Accordingly, the mass of a tetraquark is given by the expectation value of the effective Hamiltonian [15]

$$H = \sum_i m_i + H_{\text{CM}}, \quad (1)$$

where

$$H_{\text{CM}} = - \sum_{i,j} C_{ij} \lambda_i^c \cdot \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j. \quad (2)$$

The first term in Eq. (1) contains the effective masses m_i as parameters. The constants C_{ij} represent integrals in the orbital space of some unspecified radial forms of the chromomagnetic part of the one gluon-exchange interaction potential and of the wave functions.

A warning should be given to the way of determining the effective masses m_i to be used for multi-quark systems. Besides the kinetic energy contribution, they incorporate the effect of a Coulomb-like term and of the confinement, the latter still being an open problem [17]. Thus, in principle, they cannot be directly extracted from meson or baryon spectra as discussed in Appendix D. Lack of better knowledge we however use the compromise proposed in Ref. [15]

$$m_c = 1550 \text{ MeV}, \quad m_s = 590 \text{ MeV}, \quad (3)$$

but due to the arbitrariness in the choice of effective masses of quarks, precise estimates of the absolute values of tetraquark masses is difficult to make. One can have an approximate idea about the range where the spectrum should be located. But a shift of the whole spectrum is justified and sometimes even performed, like in the popular work of Maiani et al. [18], which deals with diquarks, where the arbitrariness in mass is even larger.

However, the relative distances between the eigenstates obtained from the chromomagnetic Hamiltonian (2) and the structure of its eigenstates do not depend on the effective masses, which is important for exploring the strong decay properties.

TABLE I: Theoretical and experimental meson masses in MeV

Meson	J^{PC}	Theory	Exp
J/ψ	1^{--}	3121.3	3096.9
ϕ	1^{--}	1225.9	1019.5
D_s	$0^{-?}$	2032.0	1968.5
D_s^*	$1^{-?}$	2175.7	2112.3

The parameters C_{ij} have been taken from Ref. [19] where a more complete list, containing also parameters needed in this work, is given. The required values are

$$\begin{aligned}
 C_{cs} &= 5.0 \text{ MeV}, \quad C_{c\bar{c}} = 5.5 \text{ MeV}, \\
 C_{c\bar{s}} &= 6.7 \text{ MeV}, \quad C_{s\bar{s}} = 8.6 \text{ MeV}.
 \end{aligned}
 \tag{4}$$

We should mention that the above parameters were extracted from a global fit to meson and baryon ground states. For some mesons into which $Y(4140)$ can decay in Table I we compare the experimental masses of PDG [20] with the theoretical values obtained from the two-body version of (1) and (2) in the parametrization (4) which is

$$m_{q\bar{q}} = m_q + m_{\bar{q}} - \langle \lambda_1^c \cdot \lambda_2^c \rangle \langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle C_{q\bar{q}}
 \tag{5}$$

where q stands here for any light or heavy quark. From Table I one can see that the two-body Hamiltonian (5) with the masses effective (3) systematically overestimates the meson masses. Therefore the threshold energies of the channels $J/\Psi\phi$, $D_s\bar{D}_s$, $D_s^*\bar{D}_s^*$, and $D_s^*\bar{D}_s^*$, are considerably overestimated. Due to this discrepancy it is meaningless to compare the tetraquark states with the theoretical threshold. This work questions the practice of using identical effective masses in both ordinary and exotic multiquarks. In such a case we would return us to the schematic treatment of the never observed "stable" H-dibaryon [21] predicted to be strongly bound by the chromomagnetic interaction. We do not intend to make a fine tuning of the effective masses. We are mostly interested in the structure of the tetraquark wave functions which essentially depends on the hyperfine interaction. We shall

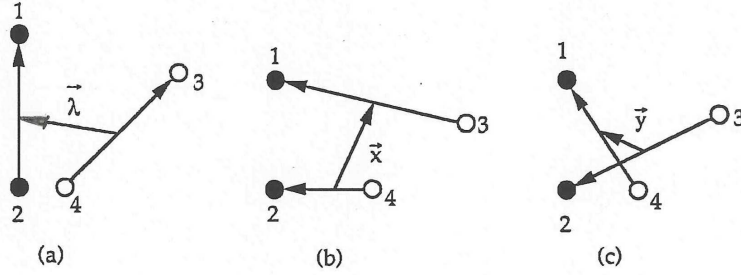


FIG. 1: Three independent relative coordinate systems. Solid and open circles represent quarks and antiquarks respectively: (a) diquark-antidiquark channel, (b) direct meson-meson channel, (c) exchange meson-meson channel.

compare the calculated spectrum to the experimental thresholds. In Appendix D we give a simple proof that one cannot use the same effective masses both in mesons and tetraquarks.

In the following, an important parameter in this study is the difference between the values of $C_{c\bar{s}}$ and C_{cs} . In fact we shall see that the replacement of the light quarks $q = u, d$ with the strange quark s does not much modify the structure of the $c\bar{c}s\bar{s}$ with respect of that of $c\bar{c}q\bar{q}$.

III. THE BASIS STATES

Here we use a basis vectors relevant for understanding the decay properties of tetraquarks. The total wave function of a tetraquark is a linear combination of these vectors. We suppose that particles 1 and 2 are quarks and particles 3 and 4 antiquarks, see Fig. 1. In principle the basis vectors should contain the orbital, color, flavor and spin degrees of freedom such as to account for the Pauli principle. But, as we consider $\ell = 0$ states the orbital part is symmetric and anyhow irrelevant for the effective Hamiltonian described in the previous section. Moreover, as the flavor operators do not explicitly appear in the Hamiltonian, the flavor part does not need to be specified. A detailed description of the three distinct bases corresponding to the three choices of internal coordinates shown in Fig. 1 is presented in Refs. [22, 23]. It was found that the inclusion of meson-meson channels accelerate the convergence, for example in $c\bar{c}q\bar{q}$ tetraquarks [24].

We remind that in the color space there are three distinct bases: a) $|\bar{3}_{12}3_{34}\rangle$, $|6_{12}\bar{6}_{34}\rangle$,

$b)$ $|1_{13}1_{24}\rangle$, $|8_{13}8_{24}\rangle$, and $c)$ $|1_{14}1_{23}\rangle$, $|8_{14}8_{23}\rangle$, associated to the three distinct internal coordinate systems shown in Fig. 1. The 3 and $\bar{3}$ are antisymmetric and 6 and $\bar{6}$ are symmetric under interchange of quarks and antiquarks respectively. This basis is convenient for diquark-antidiquark models, where usually the color space is truncated to contain only $|\bar{3}_{12}3_{34}\rangle$ states [18]. This reduces each J^{PC} spectrum to twice less states than allowed by the Pauli principle [25] and influences the tetraquark properties. The sets $b)$ and $c)$ contain a singlet-singlet color and an octet-octet color state. The amplitude of the latter vanishes asymptotically, when the mesons, into which a tetraquark decays, separate. These are called *hidden color* states by analogy to states which appear in the nucleon-nucleon problem, defined as a six-quark system [26]. The contribution of hidden color states to the binding energy of light tetraquarks has been calculated explicitly in Ref. [22]. Below we shall point out their role in the description of the of $c\bar{c}s\bar{s}$ tetraquarks. The situation is similar to the interpretation of the X(3872) resonance as a $c\bar{c}q\bar{q}$ tetraquark in Ref. [15], where its small width has been explained as due to a tiny $J/\psi + \rho$ or $J/\psi + \omega$ component in the wave function of the 1^{++} tetraquark state.

As the quarks and antiquarks are spin 1/2 particles the total spin of a tetraquark can be $S = 0$, $S = 1$ or $S = 2$.

For $S = 0$ there are two independent basis states (two Young tableaux) for each channel. The spin states associated to the three distinct internal coordinates depicted in Fig. 1 are: $a)$ $|S_{12}S_{34}\rangle$, $|\vec{A}_{12} \cdot \vec{A}_{34}\rangle$, $b)$ $|P_{13}P_{24}\rangle$, $|(V_{13}V_{24})_0\rangle$, $c)$ $|P_{14}P_{23}\rangle$, $|(V_{14}V_{23})_0\rangle$, respectively, where S stands for scalar, A for axial and P and V for pseudoscalar and vector subsystems and the lower index 0 indicates the total spin. The relation between the three different bases can be found in Ref. [23].

For $S = 1$ there are three independent spin states, corresponding to three distinct Young tableaux. Presently we are interested into those corresponding to Fig. 1b, named the direct meson-meson channel. In this channel we remind that the basis vectors are [23]

$$|(P_{13}V_{24})_1\rangle, \quad |(V_{13}P_{24})_1\rangle, \quad |(V_{13}V_{24})_1\rangle. \quad (6)$$

As above, the lower index indicates the total spin 1.

In this case the charge conjugation operator is related to permutation properties of the basis vectors in a simple way. Under the transposition (13) manifestly one has

$$(13)|P_{13}\rangle = -|P_{13}\rangle, \quad (13)|V_{13}\rangle = +|V_{13}\rangle, \quad (7)$$

and similarly for the transposition (24)

$$(24)|P_{24}\rangle = -|P_{24}\rangle, \quad (24)|V_{24}\rangle = +|V_{24}\rangle. \quad (8)$$

The case $S = 2$ is trivial. There is a single basis state

$$\chi^S = |(V_{13}V_{24})_2\rangle, \quad (9)$$

which is symmetric under any permutation of quarks.

From Ref. [27] Ch. 10, one can see that the permutation (13)(24) leaves invariant the color basis vectors $|1_{13}1_{24}\rangle$ and $|8_{13}8_{24}\rangle$. Then, with the identification $1 = c$, $2 = s$, $3 = \bar{c}$ and $4 = \bar{s}$ the permutation (13)(24) is equivalent to the charge conjugation operator [32]. Thus all basis states introduced below have a definite charge conjugation, which is easy to identify.

IV. MATRIX ELEMENTS

For a ground state tetraquark the possible states are $J^{PC} = 0^{++}, 1^{++}, 1^{+-}$ and 2^{++} . In the direct meson-meson channel, in each case a basis can be built with the quark-antiquark pairs (1,3) and (2,4) as subsystems, where each subsystem has a well defined color state, a singlet-singlet or an octet-octet. This arrangement is convenient to describe hidden charm $J/\psi + \text{light meson}$ or $\eta_c + \text{light meson}$ channels, the light meson quantum numbers being consistent with J^{PC} . The other quark-antiquark pairs, (1,4) and (2,3) describe open charm meson channels, here called exchange channels (see Fig. 1c) as *e. g.* $D_s\bar{D}_s$, $D_s\bar{D}_s^*$ or $D_s^*\bar{D}_s$. One can fix a basis in terms of the problem one looks at, but for convenience, in the calculations one can pass from one basis to another by an orthogonal transformation. In this study the adequate basis is that related to the direct meson-meson channel, depicted in Fig. 1b. The orthogonal transformations from the direct to the exchange meson-meson channel for $J^{PC} = 0^{++}$ and 1^{++} are given in Appendices A and B respectively.

The matrix elements introduced below appeared in the Proceedings [25]. For the reader's convenience we present them here again. They correspond to the scalar, axial and tensor tetraquarks introduced above. Later on, the authors of Ref. [19] calculated the matrix elements of the chromomagnetic interaction (2) in a basis corresponding to Fig. 1a. Although the spectrum is the same, one cannot distinguish between charge conjugation $C = 1$ and

$C = -1$ because in that basis $J^P = 1^+$ states do not have a definite charge conjugation. To identify C one must return to our basis. Therefore we found it convenient to use our basis which can give direct information to experimentalists.

For $J^{PC} = 0^{++}$ the basis constructed from products of color and spin states associated to Fig. 1b are

$$\begin{aligned}\psi_{0^{++}}^1 &= |1_{13}1_{24}P_{13}P_{24}\rangle, \quad \psi_{0^{++}}^2 = |1_{13}1_{24}(V_{13}V_{24})_0\rangle, \\ \psi_{0^{++}}^3 &= |8_{13}8_{24}P_{13}P_{24}\rangle, \quad \psi_{0^{++}}^4 = |8_{13}8_{24}(V_{13}V_{24})_0\rangle.\end{aligned}\tag{10}$$

The chromomagnetic interaction Hamiltonian with minus sign, $-H_{\text{CM}}$, acting on this basis leads to the following symmetric matrix

$$\begin{bmatrix} 16(C_{13} + C_{24}) & 0 & 0 & 8\sqrt{\frac{2}{3}}(C_{12} + C_{23}) \\ -\frac{16}{3}(C_{13} + C_{24}) & -8\sqrt{\frac{2}{3}}(C_{12} + C_{23}) & \frac{16\sqrt{2}}{3}(C_{23} - C_{12}) & \\ & -2(C_{13} + C_{24}) & \frac{4}{\sqrt{3}}(2C_{12} - 7C_{23}) & \\ & & \frac{16}{3}C_{12} + \frac{56}{3}C_{23} + \frac{2}{3}(C_{13} + C_{24}) & \end{bmatrix}\tag{11}$$

For $J^P = 1^{++}$ there are two linearly independent basis vectors built as products of color and the third spin state of Eq. (6) .

$$\psi_{1^{++}}^1 = |1_{13}1_{24} (V_{13}V_{24})_1\rangle, \quad \psi_{1^{++}}^2 = |8_{13}8_{24} (V_{13}V_{24})_1\rangle.\tag{12}$$

The matrix associated to the chromomagnetic interaction $-H_{\text{CM}}$ is

$$\begin{bmatrix} -\frac{16}{3}(C_{13} + C_{24}) & \frac{8\sqrt{2}}{3}(C_{23} - C_{12}) \\ & \frac{2}{3}(4C_{12} + 14C_{23} + C_{13} + C_{24}) \end{bmatrix}\tag{13}$$

which has been previously related to X(3872). Its lowest state gave a mass of 3910 MeV to X(3872) [15, 25], quite close to the experimental value [16].

For $J^P = 1^{+-}$ there are four linearly independent basis vectors built as products of color states b) and the first and second spin states of Eq. (6)

$$\begin{aligned}\psi_{1+-}^1 &= |1_{13}1_{24}(P_{13}V_{24})_1\rangle, \quad \psi_{1+-}^2 = |1_{13}1_{24}(V_{13}P_{24})_1\rangle, \\ \psi_{1+-}^3 &= |8_{13}8_{24}(P_{13}V_{24})_1\rangle, \quad \psi_{1+-}^4 = |8_{13}8_{24}(V_{13}P_{24})_1\rangle.\end{aligned}\tag{14}$$

The matrix associated to the chromomagnetic interaction $-H_{\text{CM}}$ is

$$\begin{bmatrix} 16(C_{13} - \frac{1}{3}C_{24}) & 0 & 0 & 8\sqrt{\frac{2}{3}}(C_{12} + C_{23}) \\ 0 & -\frac{16}{3}(C_{13} - C_{24}) & 8\sqrt{\frac{2}{3}}(C_{12} + C_{23}) & 0 \\ & & -2(C_{13} - \frac{1}{3}C_{24}) & -\frac{4}{3}(2C_{12} - 7C_{23}) \\ & & & \frac{2}{3}(C_{13} - 3C_{24}) \end{bmatrix}\tag{15}$$

For $J^{PC} = 2^{++}$ the basis vectors are

$$\psi_{2++}^1 = |1_{13}1_{24}\chi^S\rangle, \quad \psi_{2++}^2 = |8_{13}8_{24}\chi^S\rangle,\tag{16}$$

where χ^S is the $S = 2$ spin state (9). The corresponding $-H_{\text{CM}}$ 2×2 matrix is

$$\begin{bmatrix} -\frac{16}{3}(C_{13} + C_{24}) & -\frac{8\sqrt{2}}{3}(C_{23} - C_{12}) \\ & -\frac{2}{3}(4C_{12} + 14C_{23} - C_{13} - C_{24}) \end{bmatrix}\tag{17}$$

In the calculation of the matrix elements we have used the equalities

$$C_{14} = C_{23}, \quad C_{12} = C_{34},\tag{18}$$

due to charge conjugation.

The above matrices have been first used to calculate the full spectrum of $c\bar{c}q\bar{q}$ with $q = u, d$ [25]. They can be used in any quark model containing a chromomagnetic interaction. In that case the coefficients C_{ij} should be replaced by integrals containing the chosen form factor of the chromomagnetic interaction and the orbital wave functions of the model.

Note that the matrices (11), (13) and (17) have in common the off-diagonal matrix element $C_{23} - C_{12}$. With the identification at the end of Sec. III this leads to $C_{23} - C_{12} \equiv C_{c\bar{s}} - C_{cs}$. As $C_{c\bar{s}}$ and C_{cs} have comparable values (4) their difference is small. In the next section we shall see that this off-diagonal matrix element plays an important role in the structure of the eigenstates with $J^{PC} = 0^{++}, 1^{++}$ and 2^{++} .

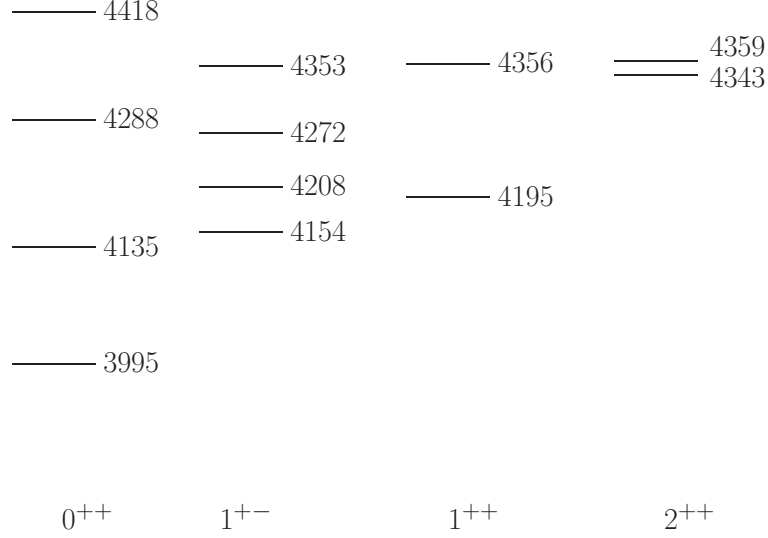


FIG. 2: The spectrum of the $c\bar{c}s\bar{s}$ tetraquark with the Hamiltonian introduced in Sec. II and the color-spin bases of Sec. III

V. THE SPECTRUM OF $c\bar{c}s\bar{s}$

The calculated spectrum is exhibited in Fig. 2. There are several states in the range 4.1 - 4.2 MeV, consistent with predictions of more realistic models [12]. This implies that the choice of the effective masses (3) is quite adequate for $c\bar{c}s\bar{s}$ tetraquarks. Here we are mostly interested in those states with a small amplitude in the VV channel in the present parametrization.

A. $\mathbf{J}^{PC} = \mathbf{0}^{++}$

In the order indicated by the basis (10) the lowest state, 3995 MeV, has the amplitudes

$$(-0.7737, 0.0594, 0.1789, 0.6049) \quad (19)$$

The first number in the bracket implies that this state can decay substantially into a PP channel, *i. e.* $\eta_c + \eta$ (threshold 3528 MeV) or $\eta_c + \eta'$ (threshold 3938 MeV). The second number indicates a very weak coupling to the VV channel. The mass is too low for the decay into $J/\psi\phi$.

A better candidate would be the first excited state at 4135 MeV with the amplitudes

$$(-0.6172, -0.1774, 0.4006, -0.6536) \quad (20)$$

decaying substantially into PP channels and much less into the VV channel $J/\psi\phi$. The last two amplitudes correspond to hidden color channels which do not decay strongly.

The tetraquark states mentioned above can also decay into the $D_s^+ D_s^-$, the threshold being 3936.68 MeV. The corresponding amplitudes can be obtained from the orthogonal transformation going from the direct meson-meson channel, Fig. 1b, to the exchange meson-meson channel, Fig. 1c. In Appendix A we present the exchange basis vectors (A2) in terms of the direct basis vectors (10) given by this transformation. Using the expressions (A3)-(A6) and the amplitudes (19) obtained in the direct channel we can write the lowest state 0^{++} in the exchange channel basis as

$$\begin{aligned} \psi_{0^{++}}(3995) = & -0.7244 |1_{14}1_{23}\rangle |P_{14}P_{23}\rangle + 0.0743 |1_{14}1_{23}\rangle |(V_{14}V_{23})_0\rangle \\ & -0.2088 |8_{14}8_{23}\rangle |P_{14}P_{23}\rangle + 0.6529 |8_{14}8_{23}\rangle |(V_{14}V_{23})_0\rangle \end{aligned} \quad (21)$$

Looking at Fig. 1c, again with $1 = c$, $2 = s$, $3 = \bar{c}$ and $4 = \bar{s}$, we can identify the color singlet-singlet channels in (A2) with the asymptotic meson-meson channels. Thus we have

$$\psi_{0^{++}}^{1ex} = |1_{14}1_{23}\rangle |P_{14}P_{23}\rangle = D_s \bar{D}_s \quad (22)$$

$$\psi_{0^{++}}^{2ex} = |1_{14}1_{23}\rangle |(V_{14}V_{23})_0\rangle = D_s^* \bar{D}_s^* \quad (23)$$

From the wave function (21) we can see that the open $D_s \bar{D}_s$ channel acquires a large amplitude in the ground state (3995 MeV), corresponding to a probability of about 50%, which will imply a large decay width in this channel and a negligible amplitude 0.5% to the closed channel $D_s^* \bar{D}_s^*$.

TABLE II: The $D_s^* - D_s$ splitting (MeV) and the amplitudes of the basis vectors (12) of the 1^{++} state at 4195 MeV as a function of $C_{c\bar{s}}$ (MeV).

$C_{c\bar{s}}$	$D_s^* - D_s$	$1_{13}1_{24}(V_{13}V_{24})_1$	$8_{13}8_{24}(V_{13}V_{24})_1$
6.0	128.0	0.0245	0.9997
6.7	143.7	0.0399	0.9992

B. $J^{PC} = 1^{+-}$

In this sector the lowest state has an appropriate mass, but not the convenient charge conjugation. It would decay exclusively into a PV channel for $\ell = 0$ tetraquarks. For general interest the exchange meson-meson basis is also given in Appendix B.

C. $J^{PC} = 1^{++}$

For the lowest 1^{++} state at 4195 MeV, which is quite close to the experimental range, the amplitudes of its components in the basis (12) are shown in Table II for two values of $C_{c\bar{s}}$. The singlet-singlet channel $1_{13}1_{24}(V_{13}V_{24})_1$ has a very small amplitude for both values of $C_{c\bar{s}}$. The hidden color state $8_{13}8_{24}(V_{13}V_{24})_1$ is by far the dominant component. The situation is entirely analogous to that of the resonance X(3872) in the same model [15, 25]. The clue was to have a nonvanishing, but small, value for $C_{23} - C_{12} \equiv C_{c\bar{q}} - C_{cq}$ in Eq. (13). For X(3872) one had 1.5 MeV, here we have $C_{c\bar{s}} - C_{cs} = 1.7$ MeV imposed by the parametrization (4). As seen from Table II a decrease of $C_{c\bar{s}}$ will make the hidden color state even more dominant but it will somewhat deteriorate the value of the $M_{D_s^*} - M_{D_s}$ splitting, the experimental value of which is 143.8 MeV. Combined with the phase space of the decay $Y(4140) \rightarrow J/\psi\phi$ obtained from the experimental threshold, the lowest state 1^{++} of the tetraquark $c\bar{c}s\bar{s}$ would acquire a rather small width, as required by experiment, and could be the best candidate for Y(4140) in a tetraquark interpretation.

It is useful to write the wave function of the lowest state also in the exchange channel basis, as for the scalar tetraquarks above. For this purpose we use the transformation between

the direct and exchange channel basis vectors derived in Appendix B, namely the Eqs. (B14) and (B15). Together with the amplitudes from Table II associated to $C_{c\bar{s}} = 6.7$ MeV we obtain for the lowest 1^{++} state

$$\psi_{1^{++}}(4195) = -0.9554 \psi_{1^{++}}^{1ex} + 0.2954 \psi_{1^{++}}^{2ex}. \quad (24)$$

From Fig 1c and Eq. (B1) one has

$$\psi_{1^{++}}^{1ex} = D_s \overline{D}_s^*, \quad \psi_{1^{++}}^{2ex} = D_s^* \overline{D}_s \quad (25)$$

According to (B8) a molecular-type component with $C = +$ is obtained in the exchange channel as

$$\psi_{1^{++}}^{1ex} = \frac{1}{\sqrt{2}} (D_s \overline{D}_s^* - D_s^* \overline{D}_s) \quad (26)$$

having a very large probability of 91.3 % in the 1^{++} ground state. The phase space is larger than for the $J/\Psi\phi$ channel, so that a large width is expected in the $D_s \overline{D}_s^*$ channel. The second term in (24) is a hidden color component, which does not decay, but vanishes asymptotically.

D. $J^{PC} = 2^{++}$

The spectrum is formed of two, nearly degenerate states, both too high for $Y(4140)$, by about 200 MeV. In the parametrization (4) the wave function of the lowest state has the amplitudes

$$(-0.4675, 0.8840) \quad (27)$$

in the order of the basis (16). One can see that the color singlet-singlet state $\psi_{2^{++}}^1 = |1_{13}1_{24}\chi^S\rangle$ has a small amplitude and the hidden color state $\psi_{2^{++}}^2 = |8_{13}8_{24}\chi^S\rangle$ is dominant, again due to the smallness of the off-diagonal matrix element $C_{c\bar{s}} - C_{cs}$. With $C_{c\bar{s}} = 6.0$ MeV the amplitudes become $(-0.2274, 0.9738)$. This would give rise to a even smaller decay width into $J/\psi\phi$. The calculated mass fits better into the newly found narrow structure $X(4350)$ reported by the Belle Collaboration [3]. According to Appendix C the wave function of the lowest state obtained from the latter amplitudes becomes

$$\psi_{2^{++}}(4343) = 0.8442 D_s^* \overline{D}_s^* - 0.5390 \psi_{2^{++}}^{2ex} \quad (28)$$

where we have replaced $\psi_{2^{++}}^{1ex}$ by its physical content. This state has a dominant molecular-type structure plus a hidden color component (C2) which would vanish asymptotically, but is important at short range. In a standard hadronic molecule interpretation [4–7] the second component is absent because the emitted mesons do not have a structure.

VI. CONCLUSIONS

Prior to the CDF experiment [1], among other multiquark systems, the tetraquark $c\bar{c}s\bar{s}$ has been studied with a different parametrization from the one considered here and with a different basis using an SU(6) classification [28]. In that basis it is difficult to identify the VV component. Moreover a distinction between charge conjugation $C = 1$ and $C = -1$ has not been made.

Our study favours mostly the 1^{++} sector for the $c\bar{c}s\bar{s}$ tetraquark interpretation of the recently observed narrow structure Y(4140) [1]. If correct, Y(4140) would be the strange analogue of X(3872), when interpreted as a $c\bar{c}q\bar{q}$ tetraquark. This observation follows from the fact that in the schematic model of Ref. [15] the chromomagnetic interaction leads to a similar composition of the wave function in the basis (12) for tetraquarks containing either u and/or d , like X(3872), or s quarks, like Y(4140).

Note however that one should consider the effective masses (3) with caution. They have been obtained from fitting baryon and meson spectra. A natural question raises whether or not these masses are adequate for tetraquarks. This study questions their use in tetraquarks, inasmuch as they contain the effect of the kinetic energy and of the confinement. The confinement has been thoroughly studied in lattice calculations. A Y-shape confinement potential is almost confirmed by lattice results (see *i. e.* [29]). Information from lattice calculations on tetraquark (see *i. e.* [30]) may lead to a better understanding of the effective masses to be used in simple models. Thus, with the present parametrization it is meaningless to look at the tetraquark spectrum relative to the theoretical threshold. A detailed discussion based on a simple example is presented in Appendix D.

Finally, we should mention that the present study does not exclude the 0^{++} sector. In fact, in the molecular $D_s^*\bar{D}_s^*$, Y(4140) can have the quantum numbers 0^{++} or 2^{++} . As mentioned in the introduction, here we stress again that the Belle Collaboration measurement of a two-photon partial width disfavors the scenario of Y(4140) to be a $D_s^*\bar{D}_s^*$ molecule with

$$J^{PC} = 0^{++} \text{ or } 2^{++} \quad [3].$$

A correct interpretation of the narrow structure $Y(4140)$ observed by CDF [1] would be possible if its existence was confirmed and its quantum numbers J^{PC} were found experimentally, in order to remove the doubt cast by some theoretical interpretations [31]. Also, the measurement of the decay widths of other open channels such as $\eta_c + \eta$ or $\eta_c + \eta'$ is important. If such decays are observed, the 0^{++} sector is favored, if not, the sector 1^{++} is favoured in a tetraquark interpretation. Complementary information can also be obtained from the decays to $D_s \overline{D}_s$, $D_s \overline{D}_s^*$, $D_s^* \overline{D}_s^*$ etc.

In conclusion, as a start, we have presented results in a tetraquark schematic model to get a hint on the interpretation of $Y(4140)$, which remains an open problem. Perhaps a more realistic view, if $Y(4140)$ was confirmed, would be to have a compact tetraquark structure at short range and a molecular structure at medium or large range. Anyhow, a more elaborate study of the $c\bar{c}s\bar{s}$ tetraquark system is worth by itself.

VII. ACKNOWLEDGMENTS

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Appendix A: Direct to exchange channel basis for $J^{PC} = 0^{++}$

In the following we need to express the color exchange basis in terms of the color direct basis vectors. The well known relations are

$$|1_{14}1_{23}\rangle = \frac{1}{3}|1_{13}1_{24}\rangle + \frac{2\sqrt{2}}{3}|8_{13}1_{24}\rangle, \quad |8_{14}8_{23}\rangle = \frac{2\sqrt{2}}{3}|1_{13}1_{24}\rangle - \frac{1}{3}|8_{13}1_{24}\rangle, \quad (\text{A1})$$

Next, using Appendix C of Ref. [23] for the spin states we obtain the spin-color exchange channel basis in terms of the spin-color direct channel basis (10).

For $J^{PC} = 0^{++}$ the exchange channel basis vectors are defined by

$$\begin{aligned} \psi_{0^{++}}^{1ex} &= |1_{14}1_{23}P_{14}P_{23}\rangle, \quad \psi_{0^{++}}^{2ex} = |1_{14}1_{23}(V_{14}V_{23})_0\rangle, \\ \psi_{0^{++}}^{3ex} &= |8_{14}8_{23}P_{14}P_{23}\rangle, \quad \psi_{0^{++}}^{4ex} = |8_{14}8_{23}(V_{14}V_{23})_0\rangle. \end{aligned} \quad (\text{A2})$$

In terms of the direct channel basis vectors (10) the orthogonal transformation is given by

the following relations

$$\psi_{0^{++}}^{1ex} = \frac{1}{6} \psi_{0^{++}}^1 - \frac{1}{2\sqrt{3}} \psi_{0^{++}}^2 + \frac{\sqrt{2}}{3} \psi_{0^{++}}^3 - \sqrt{\frac{2}{3}} \psi_{0^{++}}^4 \quad (\text{A3})$$

$$\psi_{0^{++}}^{2ex} = -\frac{1}{2\sqrt{3}} \psi_{0^{++}}^1 - \frac{1}{6} \psi_{0^{++}}^2 - \sqrt{\frac{2}{3}} \psi_{0^{++}}^3 - \frac{\sqrt{2}}{3} \psi_{0^{++}}^4 \quad (\text{A4})$$

$$\psi_{0^{++}}^{3ex} = \frac{\sqrt{2}}{3} \psi_{0^{++}}^1 - \sqrt{\frac{2}{3}} \psi_{0^{++}}^2 - \frac{1}{6} \psi_{0^{++}}^3 + \frac{1}{2\sqrt{3}} \psi_{0^{++}}^4 \quad (\text{A5})$$

$$\psi_{0^{++}}^{4ex} = -\sqrt{\frac{2}{3}} \psi_{0^{++}}^1 - \frac{\sqrt{2}}{3} \psi_{0^{++}}^2 + \frac{1}{2\sqrt{3}} \psi_{0^{++}}^3 + \frac{1}{6} \psi_{0^{++}}^4 \quad (\text{A6})$$

These relations are used to derive Eq. (21).

Appendix B: Direct to exchange channel basis for $J^{PC} = 1^{++}$

In the exchange channel corresponding to Fig. 1c the basis states can be defined as above. Note however that in this case they do not all have a definite charge conjugation. Let us first introduce the $J^{PC} = 1^+$ the exchange channel basis vectors as

$$\begin{aligned} \psi_{1^+}^{1ex} &= |1_{14}1_{23}(P_{14}V_{23})_1\rangle, & \psi_{1^+}^{2ex} &= |1_{14}1_{23}(V_{14}P_{23})_1\rangle, \\ \psi_{1^+}^{3ex} &= |1_{14}1_{23}(V_{14}V_{23})_1\rangle, & \psi_{1^+}^{4ex} &= |8_{14}8_{23}(P_{14}V_{23})_1\rangle, \\ \psi_{1^+}^{5ex} &= |8_{14}8_{23}(V_{14}P_{23})_1\rangle, & \psi_{1^+}^{6ex} &= |8_{14}8_{23}(V_{14}V_{23})_1\rangle. \end{aligned} \quad (\text{B1})$$

Using Appendix C of Ref. [23] which gives the transformations in the spin space, the exchange channel basis vectors (B1) can be written as linear combinations of the direct channel basis vectors (12) and (14). The orthogonal transformation gives the equations

$$\psi_{1^+}^{1ex} = \frac{1}{6}(\psi_{1^{+-}}^1 + \psi_{1^{+-}}^2) - \frac{1}{3\sqrt{2}} \psi_{1^{++}}^1 + \frac{\sqrt{2}}{3}(\psi_{1^{+-}}^3 + \psi_{1^{+-}}^4) - \frac{2}{3} \psi_{1^{++}}^2, \quad (\text{B2})$$

$$\psi_{1^+}^{2ex} = \frac{1}{6}(\psi_{1^{+-}}^1 + \psi_{1^{+-}}^2) + \frac{1}{3\sqrt{2}} \psi_{1^{++}}^1 + \frac{\sqrt{2}}{3}(\psi_{1^{+-}}^3 + \psi_{1^{+-}}^4) + \frac{2}{3} \psi_{1^{++}}^2, \quad (\text{B3})$$

$$\psi_{1^+}^{3ex} = -\frac{1}{3\sqrt{2}}(\psi_{1^{+-}}^1 - \psi_{1^{+-}}^2) - \frac{2}{3}(\psi_{1^{+-}}^3 - \psi_{1^{+-}}^4), \quad (\text{B4})$$

$$\psi_{1^+}^{4ex} = \frac{\sqrt{2}}{3}(\psi_{1^{+-}}^1 + \psi_{1^{+-}}^2) - \frac{2}{3} \psi_{1^{++}}^1 - \frac{1}{6}(\psi_{1^{+-}}^3 + \psi_{1^{+-}}^4) + \frac{1}{3\sqrt{2}} \psi_{1^{++}}^2, \quad (\text{B5})$$

$$\psi_{1^+}^{5ex} = \frac{\sqrt{2}}{3}(\psi_{1^{+-}}^1 + \psi_{1^{+-}}^2) + \frac{2}{3} \psi_{1^{++}}^1 - \frac{1}{6}(\psi_{1^{+-}}^3 + \psi_{1^{+-}}^4) - \frac{1}{3\sqrt{2}} \psi_{1^{++}}^2, \quad (\text{B6})$$

$$\psi_{1+}^{6ex} = -\frac{2}{3}(\psi_{1+-}^1 - \psi_{1+-}^2) + \frac{1}{3\sqrt{2}}(\psi_{1+-}^3 - \psi_{1+-}^4) . \quad (\text{B7})$$

From these relations one can see that only ψ_{1+}^{3ex} and ψ_{1+}^{6ex} have a definite charge conjugation $C = -$. But in the exchange channel one can further introduce definite charge conjugation from the basis vectors in the following way. For $C = +$ the normalized states are

$$\psi_{1++}^{1ex} = \frac{1}{\sqrt{2}} (\psi_{1+}^{1ex} - \psi_{1+}^{2ex}) , \quad (\text{B8})$$

$$\psi_{1++}^{2ex} = \frac{1}{\sqrt{2}} (\psi_{1+}^{4ex} - \psi_{1+}^{5ex}) . \quad (\text{B9})$$

Then for $C = -$ the normalized states are

$$\psi_{1+-}^{1ex} = \frac{1}{\sqrt{2}} (\psi_{1+}^{1ex} + \psi_{1+}^{2ex}) , \quad (\text{B10})$$

$$\psi_{1+-}^{2ex} = \psi_{1+}^{3ex} \quad (\text{B11})$$

$$\psi_{1+-}^{3ex} = \frac{1}{\sqrt{2}} (\psi_{1+}^{4ex} + \psi_{1+}^{5ex}) , \quad (\text{B12})$$

$$\psi_{1+-}^{4ex} = \psi_{1+}^{6ex} . \quad (\text{B13})$$

Lastly, replacing the expressions of ψ_{1+}^{1ex} , ψ_{1+}^{2ex} , ψ_{1+}^{4ex} and ψ_{1+}^{5ex} in Eqs. (B8) and (B9) we get the orthogonal transformation relating the exchange channels with the direct channel wave functions (12) for $C = +$

$$\psi_{1++}^{1ex} = -\frac{1}{3}\psi_{1++}^1 - \frac{2\sqrt{2}}{3}\psi_{1++}^2 \quad (\text{B14})$$

$$\psi_{1++}^{2ex} = -\frac{2\sqrt{2}}{3}\psi_{1++}^1 + \frac{1}{3}\psi_{1++}^2 \quad (\text{B15})$$

This transformation will be used in the subsection C of Sec. V.

Appendix C: Direct to exchange channel basis for $J^{PC} = 2^{++}$

The relations between the exchange and direct basis are in this case a direct consequence of the definitions (A1) inasmuch as the spin state χ^S is symmetric under any permutation of S_4 . One obtains

$$\psi_{2++}^{1ex} = \frac{1}{3}\psi_{2++}^1 + \frac{2\sqrt{2}}{3}\psi_{2++}^2, \quad (\text{C1})$$

$$\psi_{2++}^{2ex} = \frac{2\sqrt{2}}{3}\psi_{2++}^1 - \frac{1}{3}\psi_{2++}^2. \quad (\text{C2})$$

This transformation will be used in the subsection D of Sec. V.

Appendix D: Effective masses

First we establish the relation between effective quark masses used in these calculations and masses m_i^0 of a constituent quark model. For this purpose we start from the spin-independent part of a simple model of the commonly used type [33]

$$H_0 = \sum_i m_i^0 + \sum_i \frac{\vec{p}_i^2}{2m_i^0} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i^0} + \sum_{i<j} [V_\ell(r_{ij}) + V_C(r_{ij})] \quad (D1)$$

with a kinetic part from which the center of mass energy has been removed and a potential part containing a two-body linear confinement $V_\ell(r_{ij})$ and a Coulomb-like term $V_C(r_{ij})$

$$V_\ell(r_{ij}) = -\frac{3}{16} \lambda_i^c \cdot \lambda_j^c \left(\frac{r_{ij}}{a_0^2} - d \right), \quad V_C(r_{ij}) = -\frac{3}{16} \lambda_i^c \cdot \lambda_j^c \frac{\kappa}{r_{ij}}. \quad (D2)$$

Together with a spin-spin part identical to that of Ref. [33] (not necessary to be specified here, also used in other studies as *e. g.* Ref. [23]), we have fitted the parameters of (D1) to reproduce reasonably well the mass of J/ψ and ϕ mesons by choosing a trial wave function of the form $\phi_0 \propto \exp(-a^2 r_{ij}^2/2)$. These calculations are aimed at understanding the basic reason behind the difference between effective masses and bare masses m_i^0 . The fitted parameters are

$$m_c^0 = 1600 \text{ MeV}, \quad m_s^0 = 398 \text{ MeV}, \quad a_0 = 0.0361 \text{ MeV}^{-1/2} \text{ fm}^{1/2}, \\ d = 552.4 \text{ MeV}, \quad \kappa = 39.47 \text{ MeV fm}. \quad (D3)$$

Below they are used to estimate the expectation value of H_0 corresponding to a $c\bar{c}s\bar{s}$ system described by a trial wave function of the form $R \propto \exp[-a^2(\sigma^2 + \sigma'^2 + \lambda^2)]$, a being a variational parameter, as above. Here, for convenience, we use the internal coordinates

$$\vec{\sigma} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\sigma}' = \frac{1}{\sqrt{2}}(\vec{r}_3 - \vec{r}_4), \quad \vec{\lambda} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4). \quad (D4)$$

corresponding to Fig. 1a. Note that this function can be defined in any of the coordinate systems of Fig. 1. Using the coordinates (D4) we can work out the matrix elements of the flavor operators of (D2) in the basis $|\bar{3}_{12}3_{34}\rangle$, $|6_{12}\bar{6}_{34}\rangle$ of Sec. III. The desired expectation values obtained with the parameters (D3) are shown in Table III for all considered systems.

From these results we can define effective masses in a similar way as in Ref. [15]. We have

$$m_q^{eff} = \frac{1}{2} \langle H_0 \rangle_{q\bar{q}} \quad (D5)$$

TABLE III: Expectation values of H_0 , Eq. (D1) and of its kinetic and potential parts obtained from a trial wave function with the parameter a (see text).

System	a	Kinetic	Potential	$\langle H_0 \rangle$
	(fm ⁻¹)	(MeV)	(MeV)	(MeV)
$c\bar{c}$	2.5	229.6	-317.8	3092
$s\bar{s}$	1.4	336.7	3.27	1020
$c\bar{c}s\bar{s}$	2.1	656.8	-39.8	4477

which lead to

$$m_c^{eff} = 1546 \text{ MeV}, \quad m_s^{eff} = 510 \text{ MeV}. \quad (\text{D6})$$

Although we rely on the same PDG data [20] these masses are different from those of Eq. (3) proposed in Ref. [15]. The difference is however very small for the c quark and this can be explain by the cancellation of the kinetic and potential energies, as one can see from Table III. Such a cancellation does not take place for the quark s . Thus in a dynamical approach based on a Hamiltonian like (D1) there is a cancellation of various parts of the Hamiltonian. The cancellation is more subtle in a tetraquark which has 6 distinct quark-quark or quark-antiquark pairs, while in a meson there is only one pair. It follows then that the effective masses needed for a tetraquark can be different from those of Eq. (D6). Indeed, using Table III, we obtain

$$m_c^{eff} + m_s^{eff} = \frac{1}{2} \langle H_0 \rangle_{c\bar{c}s\bar{s}} = 2238.5 \text{ MeV} \quad (\text{D7})$$

which is different from the sum of masses in (D6). This proves that one cannot use the same effective masses in mesons and tetraquarks. In this light we can consider the choice (3) acceptable and understand why the agreement with the experiment in Table I is unsatisfactory for mesons. A better knowledge of the confinement and more precise calculations

are necessary to obtain the mass of the $c\bar{c}s\bar{s}$ tetraquark relative to the $J/\psi\phi$ threshold.

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